Black Swan Events
What Can We Learn From Derivatives Data?

Philippe Henrotte - ITO 33 & HEC Paris

Volatility Investing

London 6 April 2017

UK Parliamentary Commission on Banking Standards - June 2013

"We actually got an external advisor [to assess how frequently a particular event might happen] and they came out with one in 100,000 years and we said no, and I think we submitted one in 10,000 years. But that was a year and a half before it happened. It doesn't mean to say it was wrong; it was just unfortunate that the 10,000th year was so near"
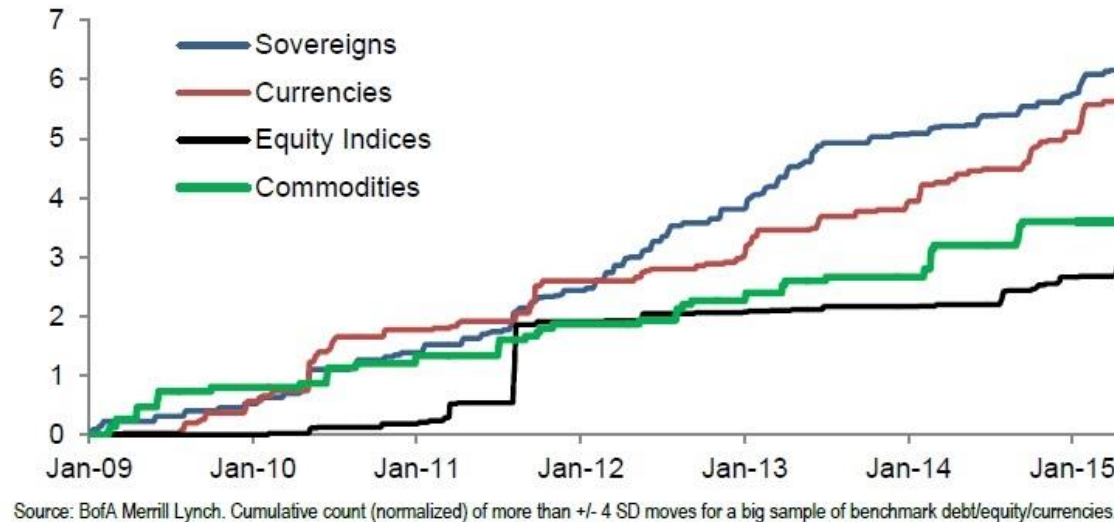
Volatility Swings

"Upheavals that shatter prevailing calm in equities are becoming more frequent. Markets were shocked out of one of the longest stretches of tranquility ever recorded last month when $600 billion was erased from the value of shares on Sept. 9. In the last two years there have been five similarly extreme shifts in volatility - as many as occurred in the prior two decades"

Joseph Ciolli, Bloomberg, October 2016

Correction Counter

Chart 1: The "correction counter". More assets are registering big standard deviation moves as central banks continue to dominate the market narrative.

Legend:
- Sovereigns
- Currencies
- Equity Indices
- Commodities

Source: BofA Merrill Lynch. Cumulative count (normalized) of more than +/- 4 SD moves for a big sample of benchmark debt/equity/currencies.

"The number of assets registering large moves—four or more standard deviations away from their normal trading range—has been growing. Our back-of-the-envelope scribblings pin that kind of move as something that might be expected to happen every 62 years"

Tracy Alloway, Bloomberg, March 2015

Scary Tails

- S&P 500 on 24 March 2017

- 3 month European Put 75% OTM Forward

- Market value: 2 dollars

- Implied volatility: 28%

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Scary Tails

- S&P 500 on 24 March 2017

- 1 month European Put 50% OTM Forward

- After careful calibration accounting for large rare negative jumps: model value 1 cent

- With 28% volatility diffusion: value is 300,000,000 smaller

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Equity Derivatives

- Equity indices such as S&P 500 or Euro Stoxx 50 offer rich families of liquid derivatives with maturities up to 5 years

- We need robust and versatile models to jointly calibrate all available market quotes

- The model should be rich enough to allow for rare and catastrophic events

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Stochastic Structure of Jumps

- The critical part is not just the jumps in the underlying but their stochastic behavior

- The dynamics of the jumps creates a stochastic skew

- Complex correlation structure with the underlying price, its volatility, and credit for single names

- An important component of the vol of vol
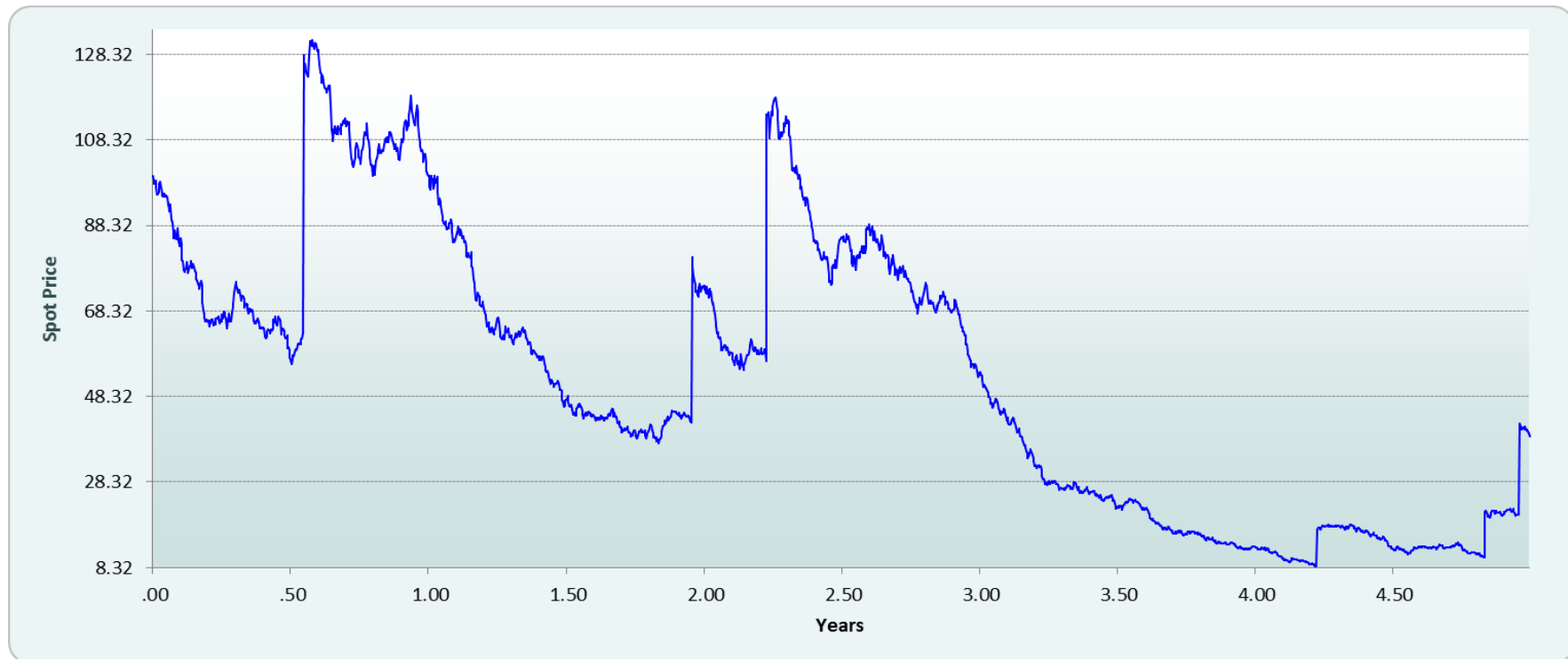
Regime Switching Model
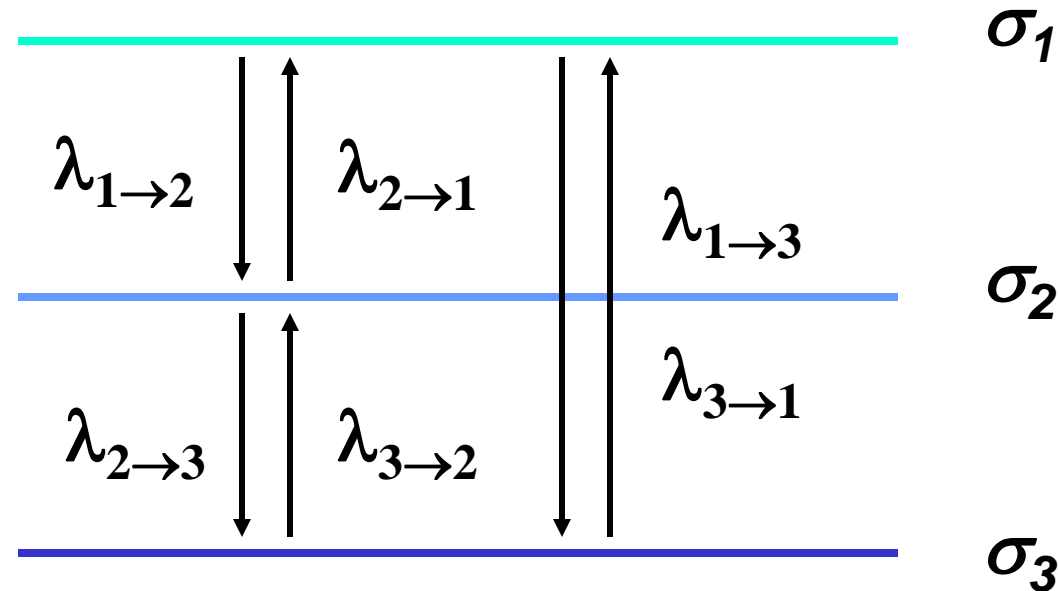
- Continuous time Markov transitions
- Underlying may jump between regimes
- Time and spot homogeneity

VIX 5 Year Path

VIX
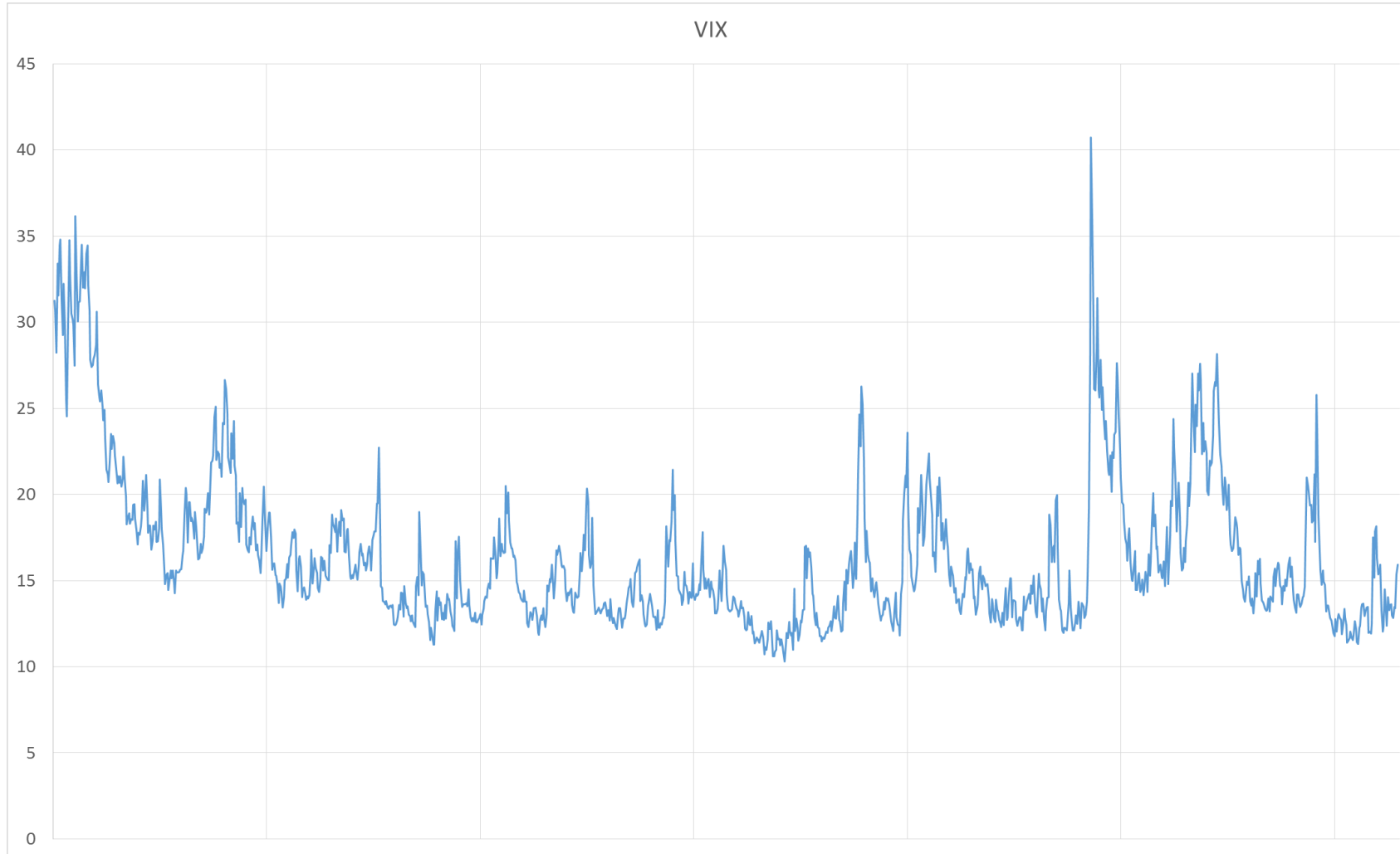
Basic Jump Diffusion Path

- Brownian volatility 30%
- One jump with intensity 1 and size +100%
- Negative drift compensates the jump in the risk neutral distribution
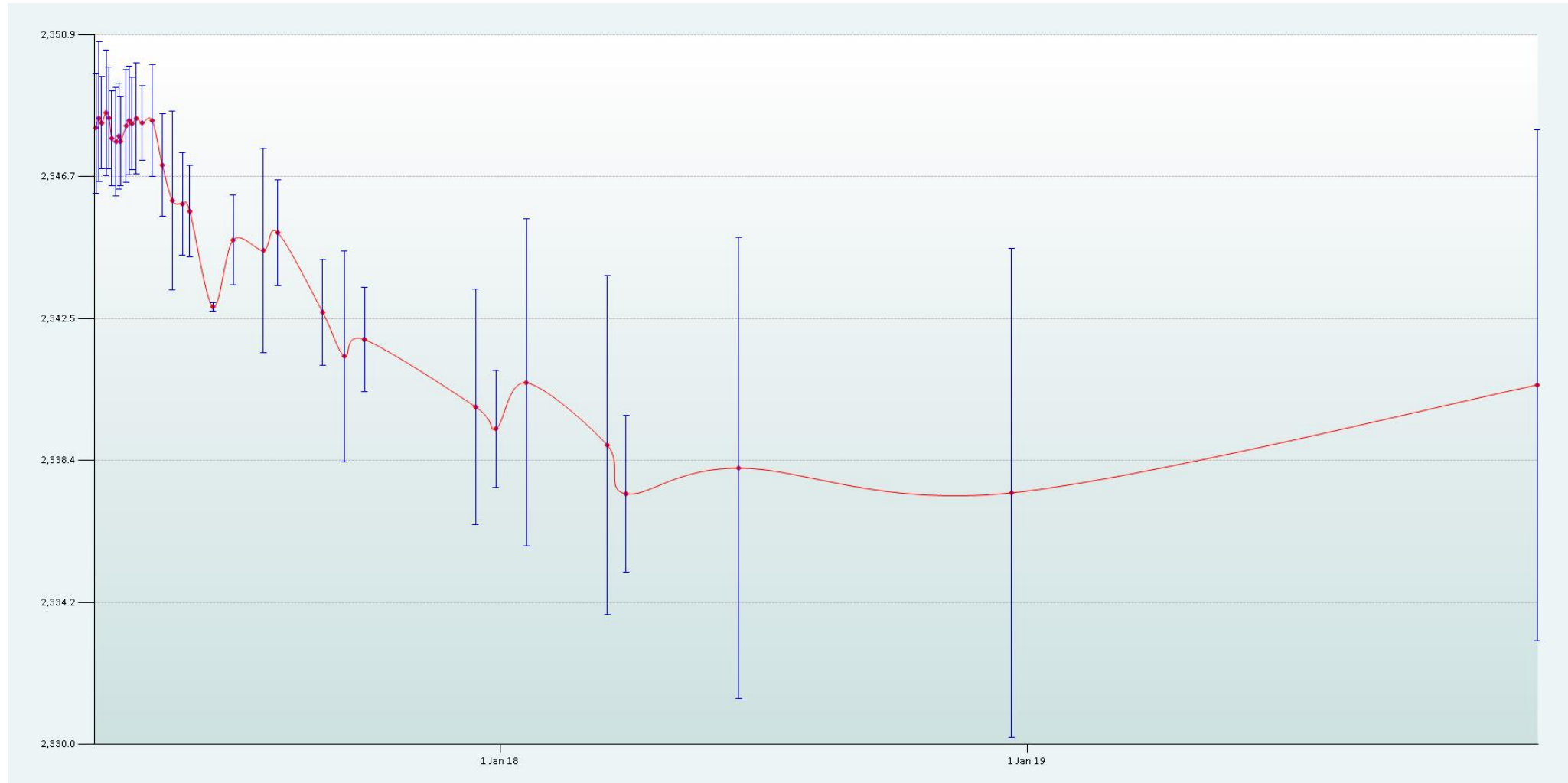
SPX Calibration
24 March 2017

Calibration Universe

- **Delta one**
  - ❑ 34 futures and synthetic forwards up to Dec 2019
- **Smile**
  - ❑ 4,050 OTM vanilla options up to Dec 2019
- **VIX**
  - ❑ 12 futures up to Nov 2017
  - ❑ 561 options up to Sept 2017

Calibration

- **Delta one calibration**
  - Index value
  - Dividend yield schedule
  - Withholding tax
  - Borrow rate schedule
- **Volatility calibration**
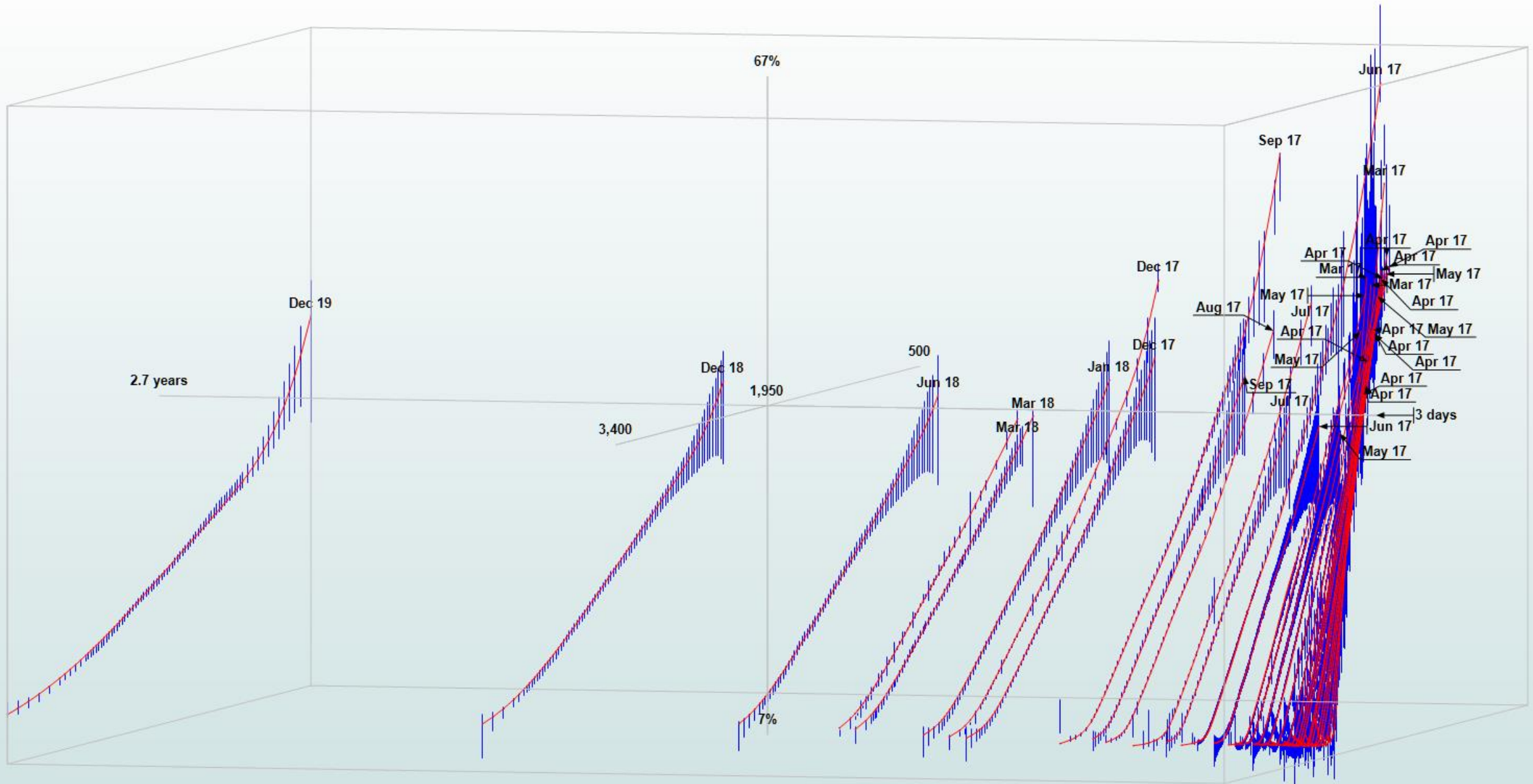 - 5 Regime volatilities
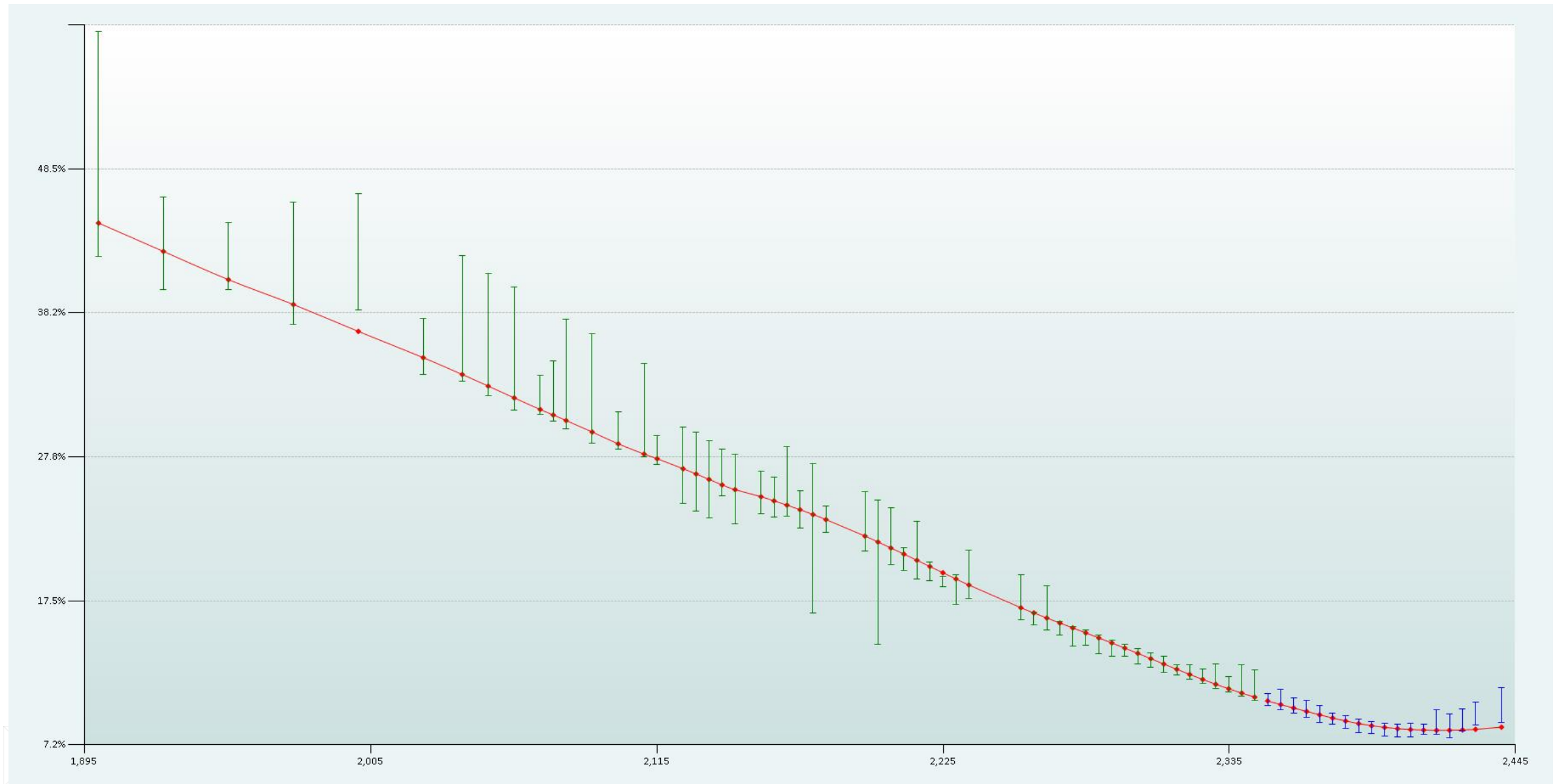 - Jumps between regimes

Futures & Synthetic Forwards

Volatility Smile

Smile 3 April 2017

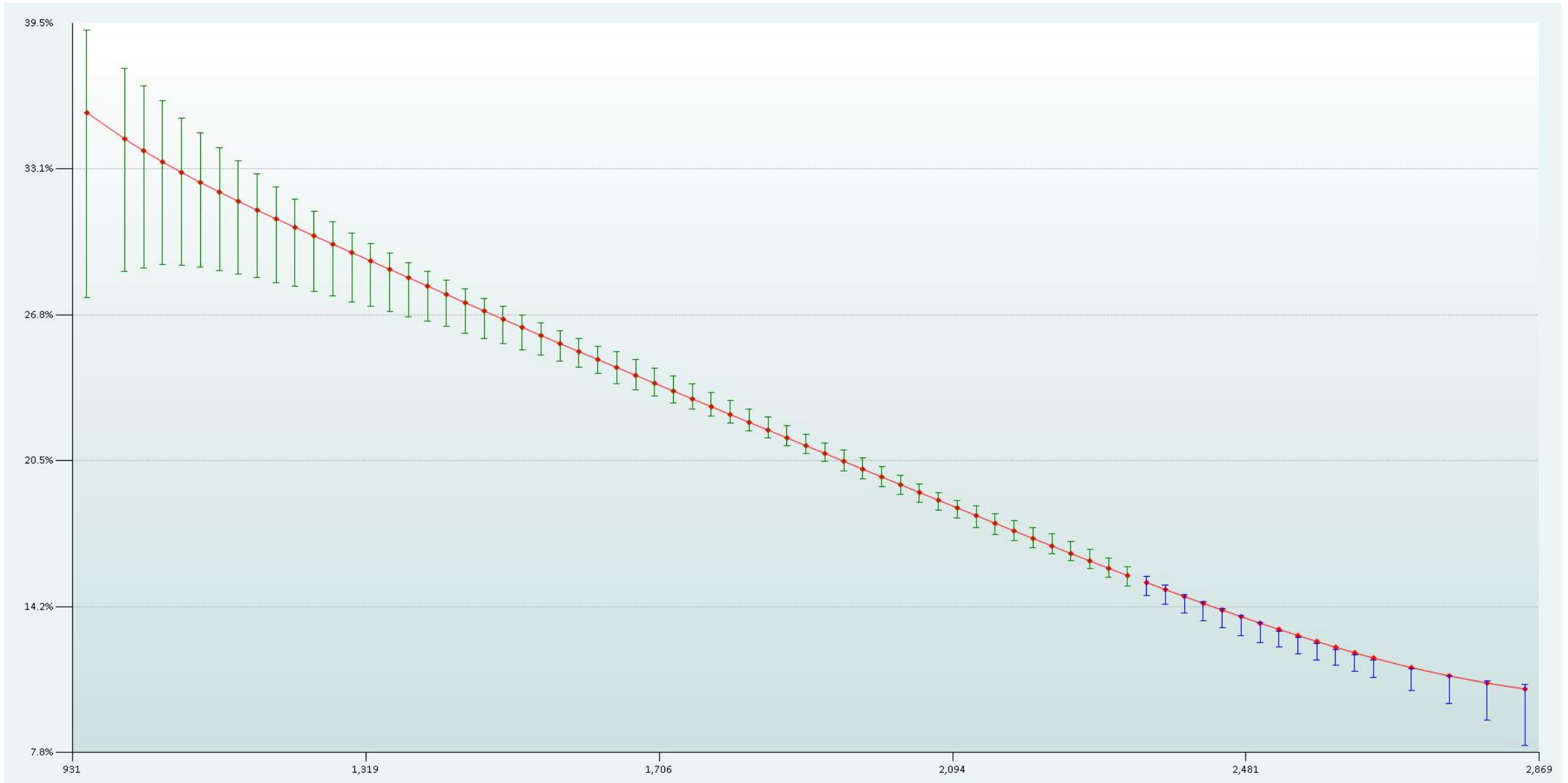# Smile 15 June 2018

Smile 20 December 2019
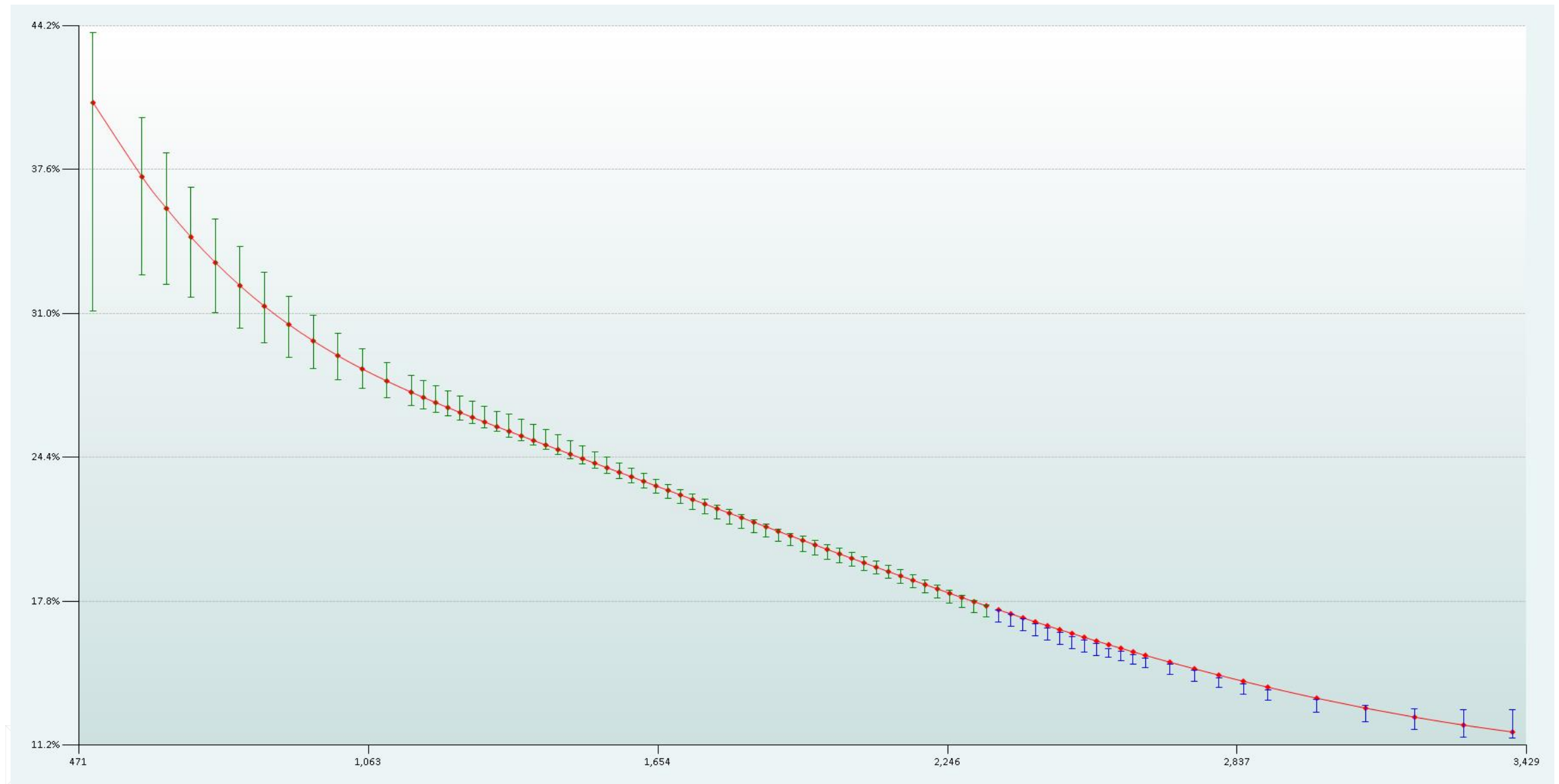
VIX Futures
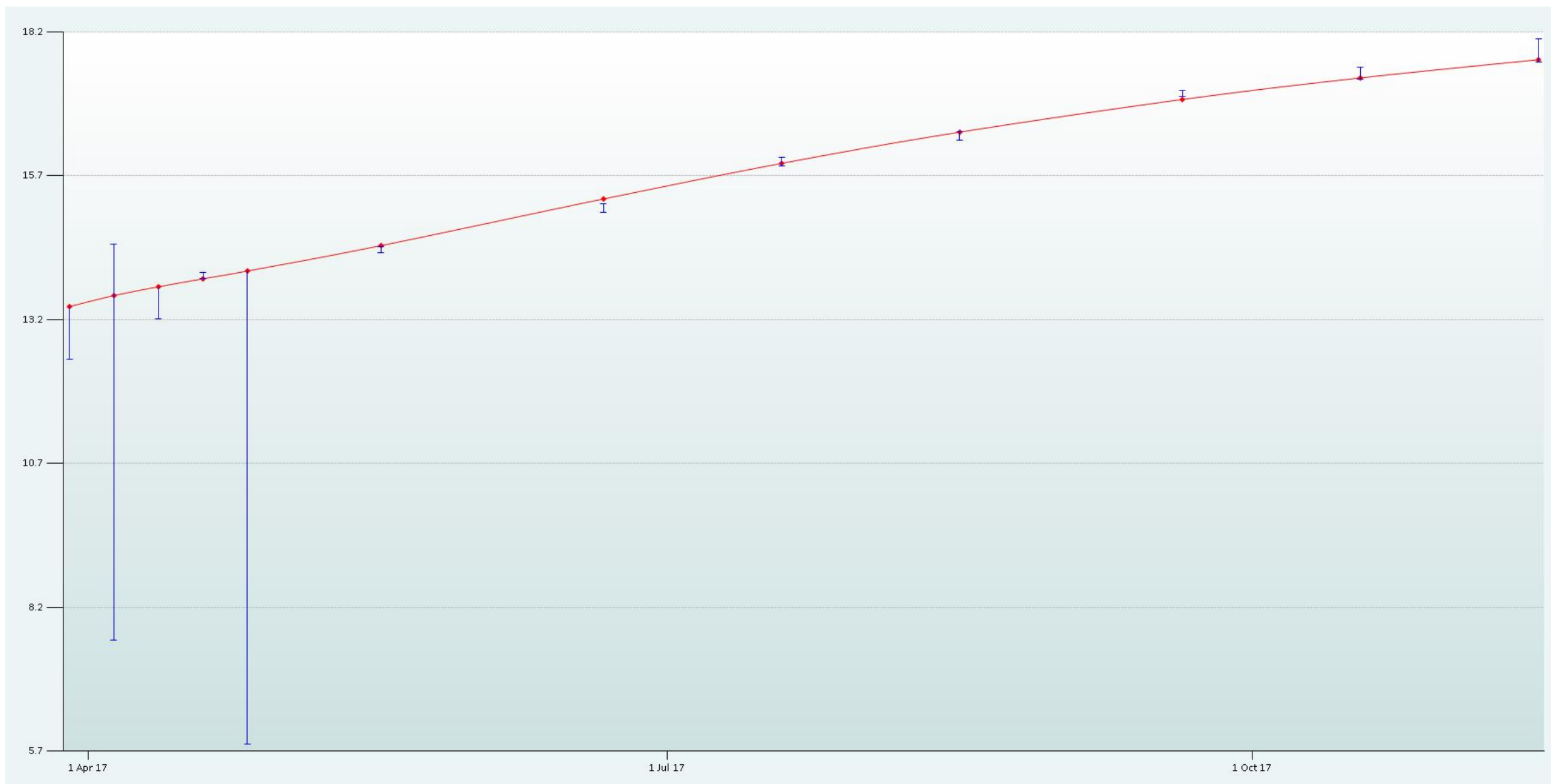
VIX Smile 12 April 2017
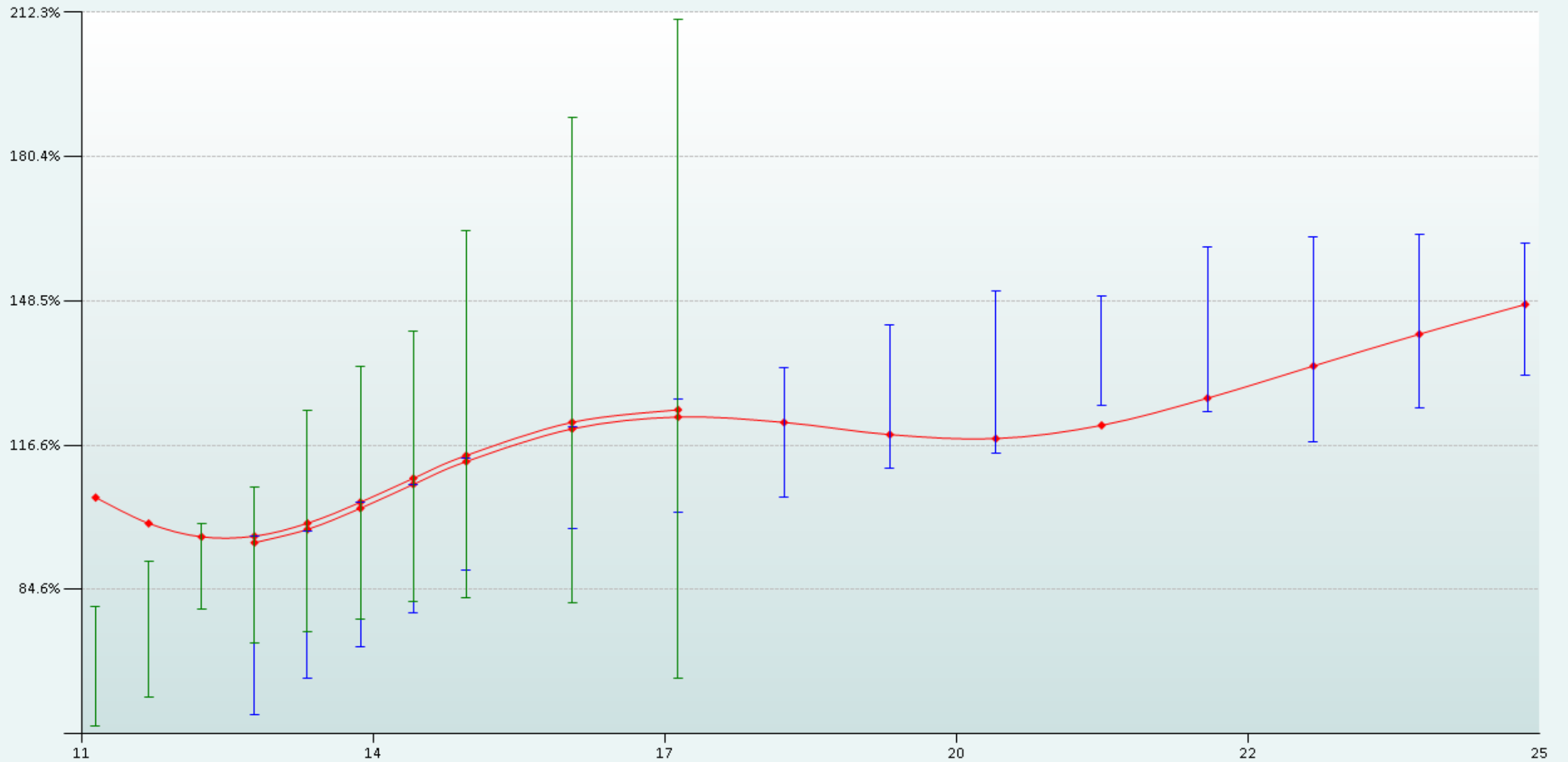
Joint Calibration
5 Regimes

	Volatility	Total Vol
Regime 1	7.21%	12.13%
Regime 2	1.00%	5.32%
Regime 3	16.85%	40.37%
Regime 4	8.84%	17.90%
Regime 5	1.00%	10.66%

|	Intensity	Size
Regime 1->2	0.416	-9.35%
Regime 1->3	0.000	0.00%
Regime 1->4	4.327	-3.28%
Regime 1->5	10.000	1.11%
Regime 2->1	5.824	-2.17%
Regime 2->3	0.000	0.00%
Regime 2->4	0.000	0.00%
Regime 2->5	0.000	0.00%
Regime 3->1	0.162	-43.35%
Regime 3->2	0.016	-97.65%
Regime 3->4	0.226	-24.07%
Regime 3->5	1.401	23.27%
Regime 4->1	0.172	-10.36%
Regime 4->2	1.588	6.57%
Regime 4->3	0.727	-14.61%
Regime 4->5	0.000	0.00%
Regime 5->1	0.000	0.00%
Regime 5->2	10.000	0.34%
Regime 5->3	0.075	-35.53%
Regime 5->4	4.889	-1.83%

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Gap Options

- Pays off and expires at the first daily positive or negative gap
- Capped at 20%, quoted in bp for a nominal of 1

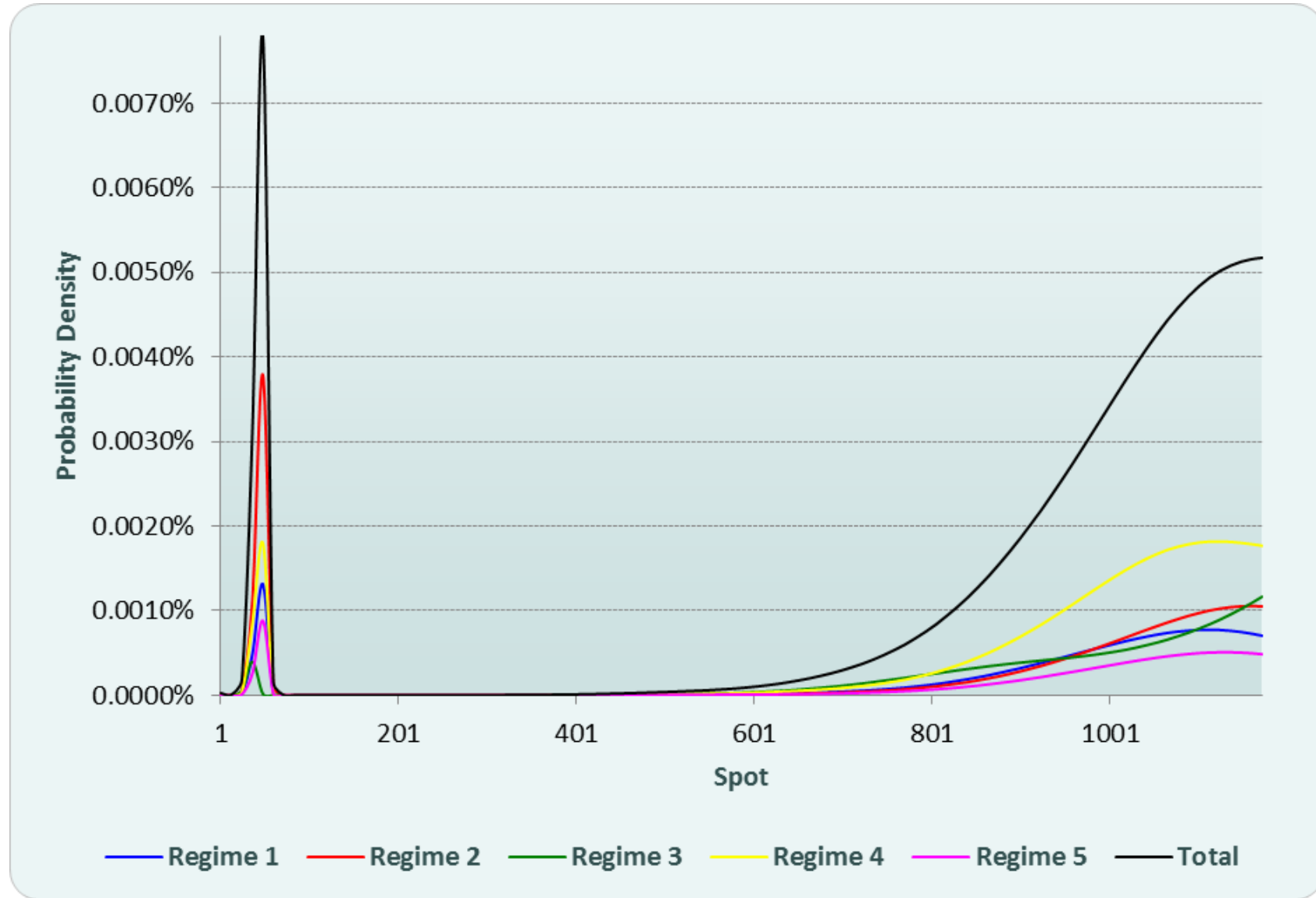| Code	Expiry	-25%	-20%	-15%	+15%	+20%	+25%
15d	8 Apr 17	0.4	0.6	0.9	0.0	0.0	0.0
1m	24 Apr 17	1.4	2.0	2.8	0.3	0.1	0.0
3m	24 Jun 17	5.1	7.3	9.5	5.4	2.1	0.0
6m	24 Sep 17	12.4	17.5	20.7	24.7	9.7	0.1
1y	24 Mar 18	34.0	46.9	49.9	88.1	34.5	0.3
2y	24 Mar 19	85.8	116.1	114.2	234.1	91.8	0.8
5y	24 Mar 22	231.9	299.7	262.6	525.7	206.3	2.3

A Random Path
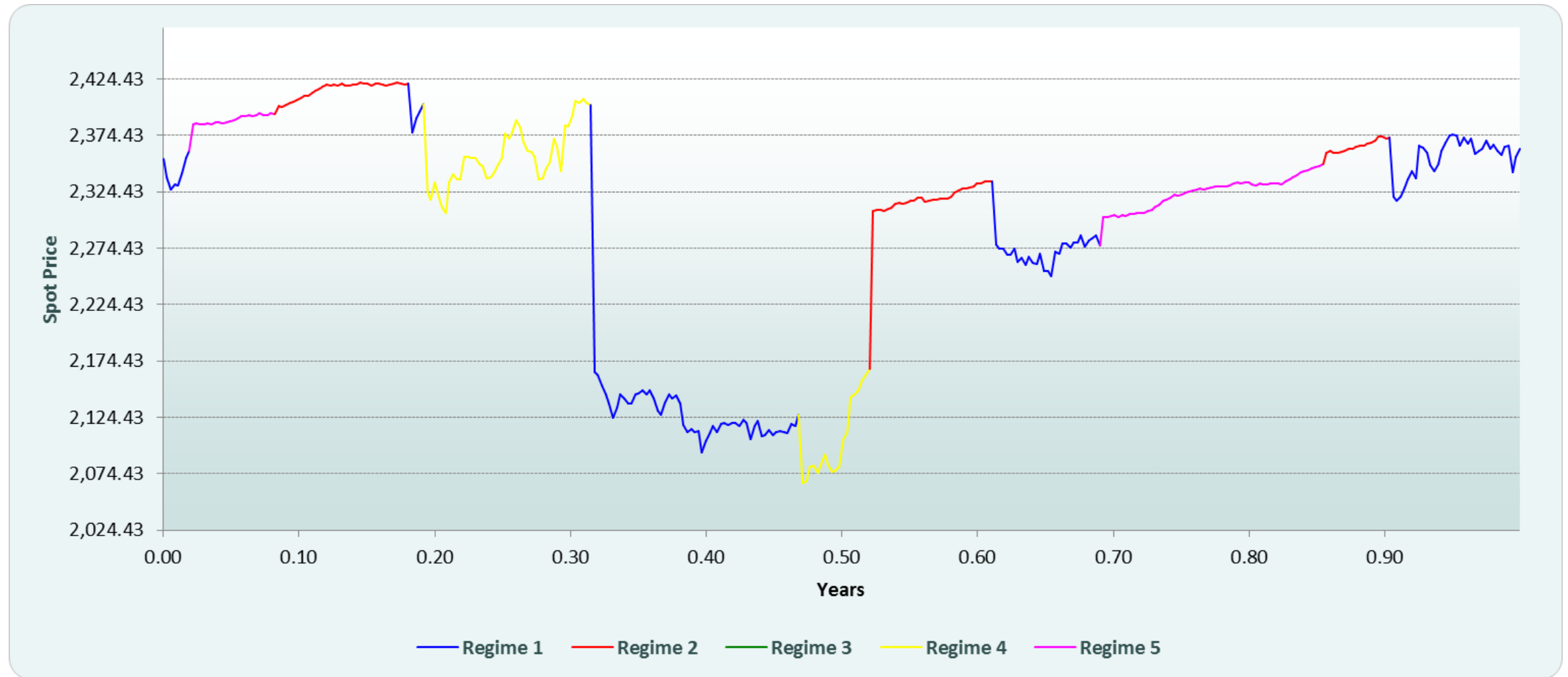
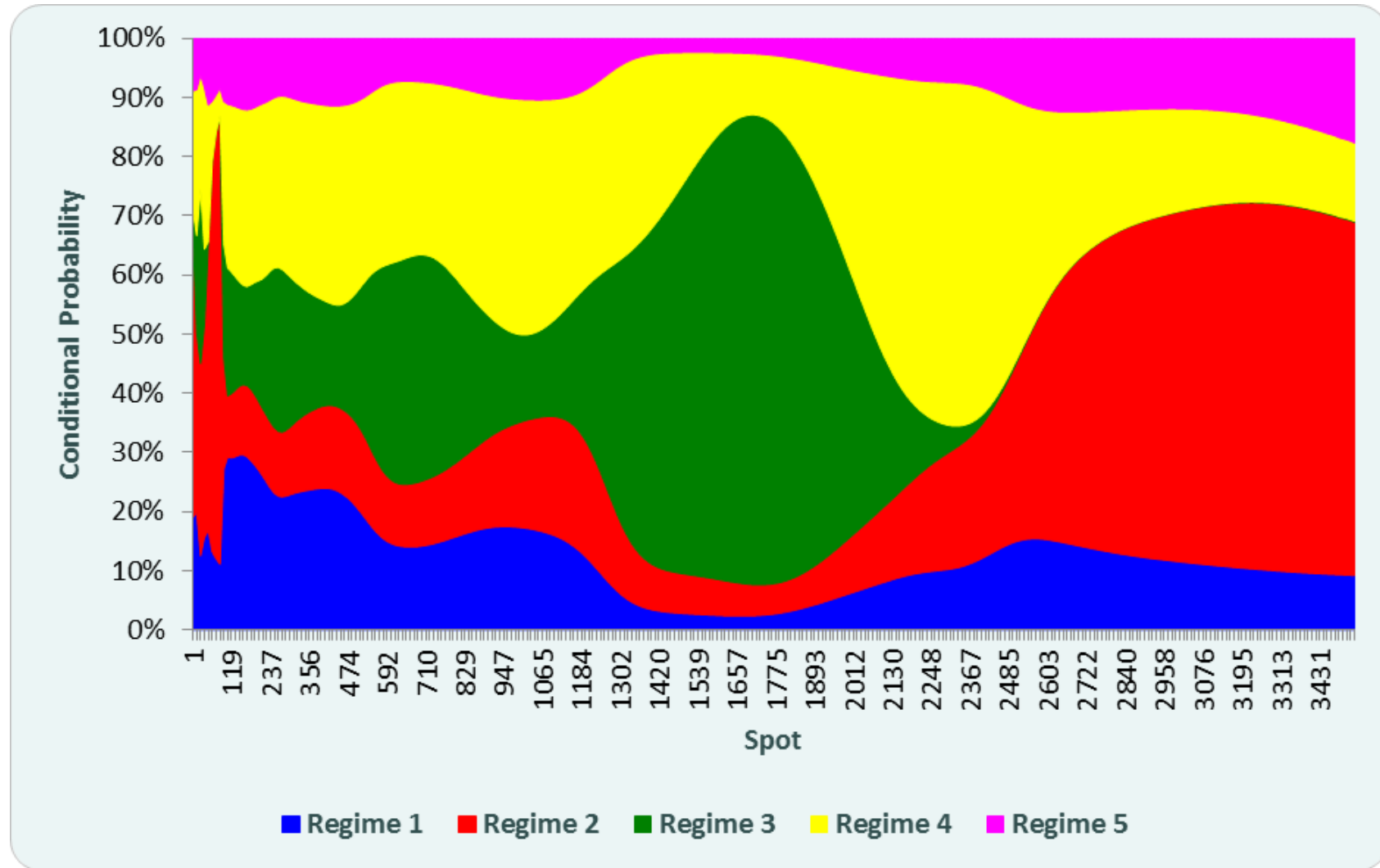# Risk Neutral Probability Densities After 1 Year
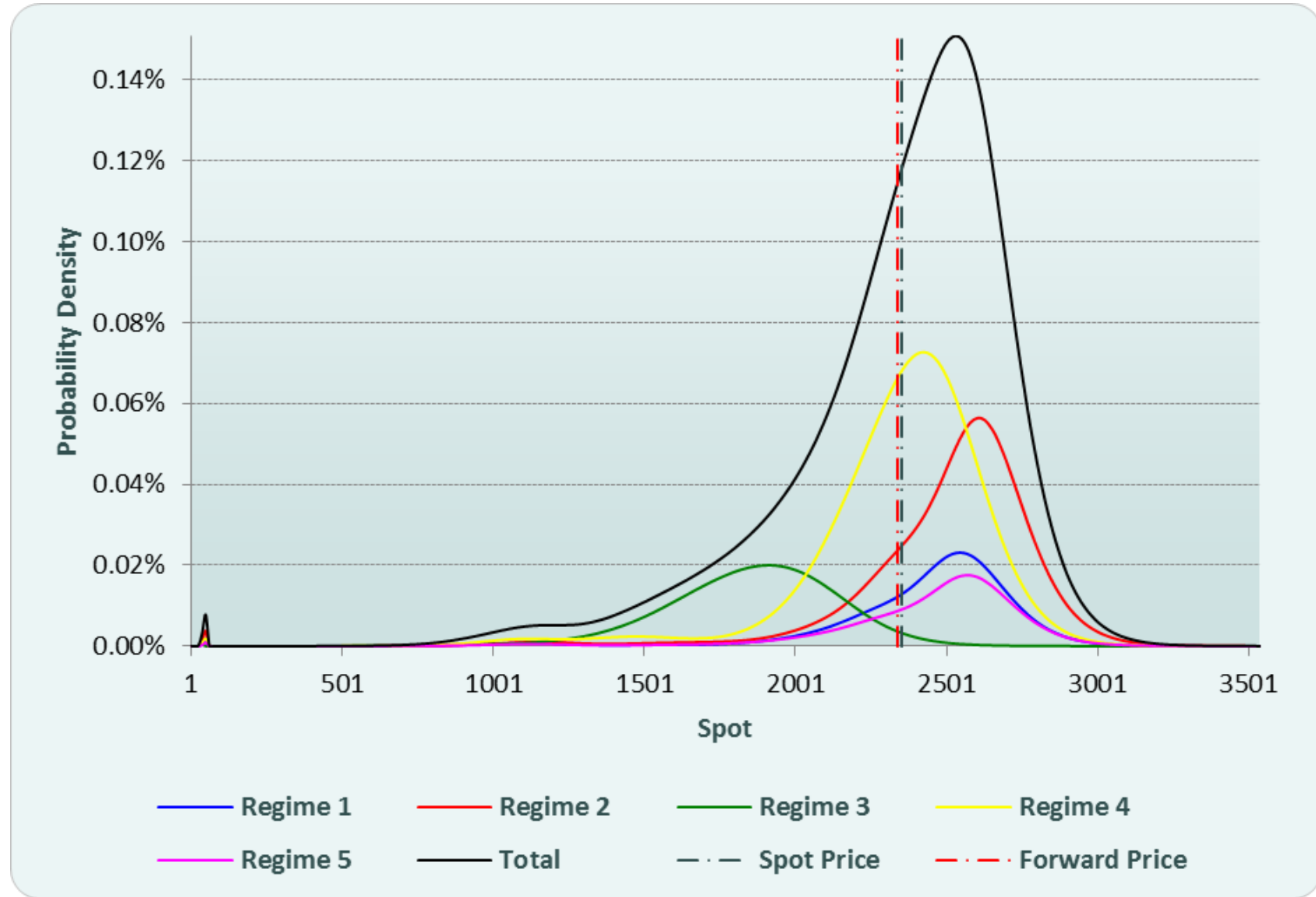
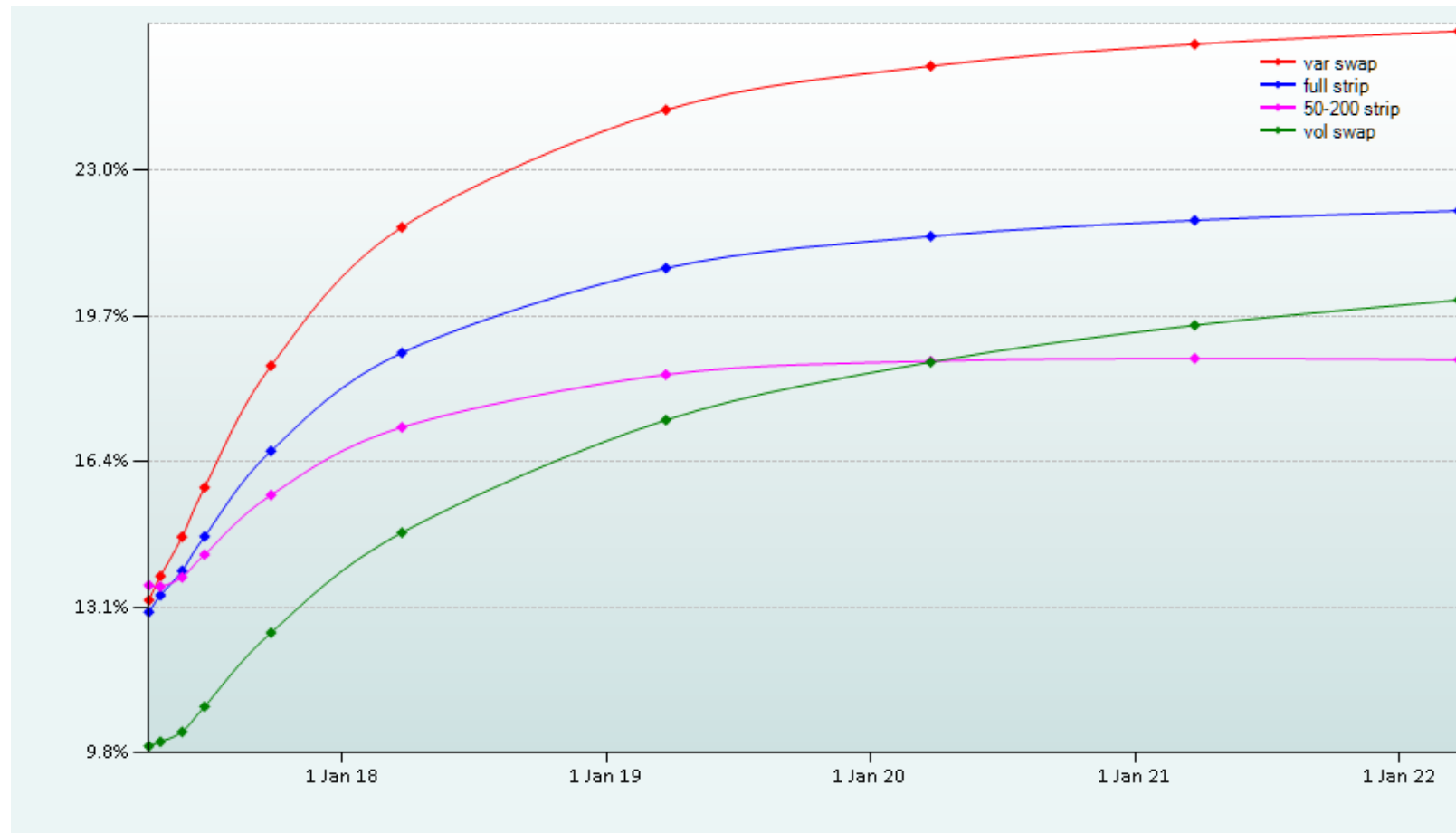# Left Tail of Probability Densities after 1 Year

Risk Neutral Probability Densities After 3 Years
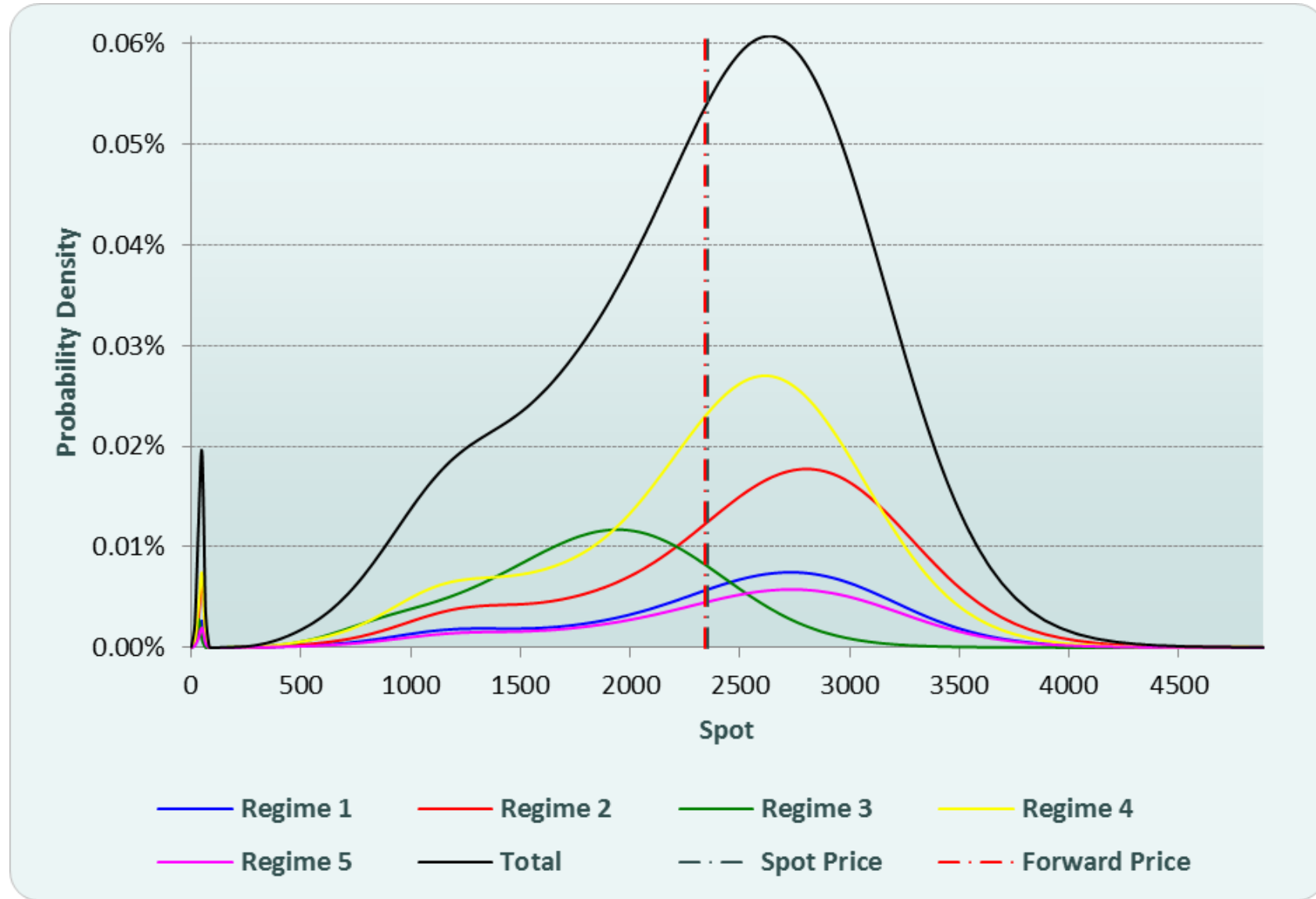
Conditional Probabilities After 1 Year

Variance Swap Analysis

Code	15d	1m	2m	3m	6m	1y	2y	3y	4y	5y
Expiry	8 Apr 17	24 Apr 17	24 May 17	24 Jun 17	24 Sep 17	24 Mar 18	24 Mar 19	24 Mar 20	24 Mar 21	24 Mar 22
Var Swap	13.28	13.83	14.71	15.83	18.58	21.71	24.36	25.35	25.84	26.14
Full Strip	13.02	13.39	13.95	14.72	16.65	18.87	20.78	21.50	21.86	22.08
50-200 Strip	13.62	13.59	13.80	14.31	15.66	17.19	18.38	18.68	18.74	18.71
Vol Swap	9.99	10.09	10.31	10.88	12.55	14.81	17.35	18.66	19.49	20.06